UM-P -016-2002

Magnetic Properties of Scalar Particles — the Scalar Aharonov-Casher Effect and Supersymmetry

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(31 July 2002)

Abstract

The original topological Aharonov-Casher (AC) effect is due to the interaction of the anomalous magnetic dipole moment (MDM) with certain configurations of electric field. Naively one would not expect an AC effect for a scalar particle for which no anomalous MDM can be defined in the usual sense. In this letter we study the AC effect in supersymmetric systems. In this framework there is the possibility to deducing the AC effect of a scalar particle from the corresponding effect for a spinor particle. In 3+1 dimensions such a connection is not possible because the anomalous MDM is zero if supersymmetry is an exact symmetry. However, in 2+1 dimensions it is possible to have an anomalous MDM even with exact supersymmetry. We then compute the anomalous MDM at the one loop level, showing how the scalar form arises in 2+1 dimensions from the coupling of the scalar to spinors. The AC effect corresponding to a scalar can be uniquely identified. This model shows us how an AC effect for a scalar can be generated for non-supersymmetric theories.

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The study of topological phases has provided a deep understanding of quantum systems. A particularly interesting case of a topological phase is the Aharonov-Bohm (AB) effect, discovered in 1959 by Aharonov and Bohm [1]. The AB effect has been observed experimentally [2]. In 1984 Aharonov and Casher discovered [3] another configuration where a topological phase can develop, giving rise to what is now called the Aharonov-Casher (AC) effect. This effect has also been observed experimentally [4]. The original AC effect was for a particle with spin and a non-zero anomalous magnetic dipole moment (MDM) interacting with a two dimensional electric field perpendicular to the spin polarisation direction. It was realised that spin 1/2 is a particularly simple and instructive case, but the AC effect and other related effects have also been studied for particles with different spins [5–10].

For a spin-1/2 particle the interaction responsible for the AC effect is given by the following anomalous MDM interaction,

$$L_m = -\frac{1}{2}\mu\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}. \tag{1}$$

The topological phase θ_{AC} developed when the particle travels along a closed path that encircles a line charge of strength Λ per unit length is $\theta_{AC} = \mu\Lambda$

In general it is possible for a particle with a non-zero anomalous MDM to have an AC effect. One would naively think that there should be no such an effect for a spin-0 particle because there there is no MDM, anomalous or otherwise. This naive expectation may not hold in 2+1 dimensions, where the AC effect can be interpreted as an interaction of a current with the dual electric field [10]. If one makes a similar extension to a spin-0 particle, an AC like effect can be defined for a spin-0 particle. This is a very suggestive way of identifying new topological effects. However one still can not be sure that the topological nature of the spin-0 case has the same origin as the spin-1/2 case. In this paper we address the question of whether a spin-0 particle can have an AC effect from the point of view of supersymmetry. We find indeed that a spin-0 particle can have an AC effect in 2+1 dimensions. This will enable us to discuss the AC effect for a spin zero particle in more general situations.

In supersymmetry, the spinor (spin-1/2) and the scalar (spin-0) particles are partners. If certain effects exist for a spinor, there should be corresponding effects for the scalar superpartner. Therefore one would expect that there must be an AC effect for a spin-0 particle. This turns out to be not automatically true. In fact it was shown some time ago that in 3+1 dimensions, if supersymmetry is exact, no anomalous MDM can exist for a spinor [11]. Of course, in nature supersymmetry is broken, so there can be a non-zero anomalous MDM for a spinor and therefore an AC effect. The corresponding effect for the scalar superpartner need not exist because supersymmetry is broken. However this result holds only in 3+1 dimensions. In 2+1 dimensions with supersymmetry, a spinor can have an anomalous MDM and therefore the associated AC effect. Because supersymmetry is exact, one can uniquely identify the corresponding AC effect for the scalar superpartner.

In 3+1 dimensions with supersymmetry, a matter field is assigned to a chiral superfield Φ which contains the spinor ψ and scalar ϕ as component fields [12]

$$\Phi = \phi(z) + \sqrt{2}\theta\psi(z) + \theta\theta F(z), \tag{2}$$

where $z_\mu = x_\mu + i\theta\sigma_\mu\theta$. θ is the anti-symmetric spinor coordinates of the superfield. F is an auxiliary field which can be eliminated by the use of the equations of motion.

The gauge superfield V in the Wess-Zumino gauge is given by [12]

$$V = -\theta\sigma_\mu\bar{\theta}A^\mu(x) + i\theta\theta\bar{\theta}\bar{\lambda}(x) - i\bar{\theta}\bar{\theta}\theta\lambda(x) + \frac{1}{2}\theta\theta\bar{\theta}\bar{\theta}D(x), \tag{3}$$

where A^μ is the usual gauge field, λ is the gaugino field and D is an auxiliary field.

The usual anomalous MDM interaction is contained in the superfield Lagrangian [11]

$$\tilde{L}_m = ig\Phi^* D_\alpha \Phi W^\alpha, \tag{4}$$

where $D_\alpha = \partial/\partial\theta^\alpha + i\sigma^\mu_{\alpha\dot{\alpha}}\bar{\theta}^{\dot{\alpha}}\partial_\mu$, $W^\alpha = (-1/4)\bar{D}\bar{D}D^\alpha V$ with $\bar{D}_{\dot{\alpha}} = -\partial/\partial\bar{\theta}^{\dot{\alpha}} - i\theta^\alpha\sigma^\mu_{\alpha\dot{\alpha}}\partial_\mu$.

Expanding $\tilde{L}_m$ in terms of θ, one would obtain [11]

$$\tilde{L}_m = E + \chi\theta + \bar{\psi}\bar{\theta} + M\theta\theta + J_\mu\theta\sigma^\mu\theta + \bar{\xi}\bar{\theta}\theta\theta. \tag{5}$$

The anomalous MDM interaction can be contained only in the M term. This can be seen easily from a dimensional analysis argument. The anomalous MDM interaction is a dimension 5 operator composed of fields and derivatives. Inspection of $\tilde{L}_m$, shows that only the M term is of dimension 5. However the superfield Lagrangian $\tilde{L}_m$ is not a chiral field, so the M term, even being a F-term type, can not appear in the supersymmetric Lagrangian density. There is no way one can obtain a supersymmetric anomalous MDM in 3+1 dimensions.

In 2+1 dimensions the situation changes dramatically. It is possible to have a supersymmetric anomalous MDM. In this case the matter field Φ, the gauge field, and the corresponding W^α are given by

$$\begin{aligned}
&\Phi(x_\mu, \theta) = \phi(x) + \theta^\lambda\psi_\lambda(x) - \frac{1}{2}\epsilon_{\lambda\tau}\theta^\lambda\theta^\tau F(x), \\
&V^\alpha(x_\mu, \theta) = i\theta^\beta(\gamma^\mu A_\mu(x))^\alpha_\beta - \epsilon_{\lambda\tau}\theta^\lambda\theta^\tau\lambda^\alpha(x), \\
&W^\alpha = \frac{1}{2}D^\beta D^\alpha V_\beta, \quad D_\alpha = \frac{\partial}{\partial\theta^\alpha} + i\theta^\beta\epsilon_{\alpha\delta}(\gamma^\mu\partial_\mu)^\delta_\beta.
\end{aligned} \tag{6}$$

Here $\epsilon_{\alpha\beta}$ is the totally anti-symmetric tensor with $\epsilon_{12} = 1$.

In 2+1 dimensions, the anomalous MDM interaction has dimension 7/2. The superfield Lagrangian which contains the anomalous MDM in 2+1 dimensions, given by

$$\tilde{L}_m = \Phi^* W_\alpha \Delta^\alpha \Phi, \tag{7}$$

(with $\Delta^\alpha = D^\alpha - ieV^\alpha$), has dimension 5/2. Expanding in terms of the θ, we have

$$\tilde{L}_m = A + B\theta + C\theta\theta. \tag{8}$$

The C term has dimension 7/2 which has the right dimension for the anomalous MDM interaction and is also supersymmetric. Therefore in 2+1 dimensions it is possible to have an anomalous MDM interaction and also the associated AC effect. Since the interaction is supersymmetric, it is possible to identify uniquely the scalar AC effect. In the following we provide the detailed calculation to obtain the form of scalar AC effect interaction.

The Lagrangian for the gauge and matter kinetic energies, and gauge and matter interactions is given by

$$L = \int d^2\theta [\frac{1}{4} W^\alpha W_\alpha + \frac{1}{2}(\Delta^\alpha \Phi)^*(\Delta_\alpha \Phi) - m\Phi^*\Phi]. \tag{9}$$

From the above we obtain

$$L = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}i\bar{\lambda}\gamma^\mu\partial_\mu\lambda + \bar{\psi}i\gamma^\mu D_\mu\psi - m\bar{\psi}\psi + (D^\mu\phi)^*(D_\mu\phi) + L_F$$
$$+ ie\bar{\psi}\lambda\phi - ie\bar{\lambda}\psi\phi^*. \tag{10}$$

Here $D^\mu = \partial^\mu - ieA^\mu$ is the gauge covariant derivative. L_F contains the F term and is given by

$$L_F = -F^*F - mF^*\phi + m\phi^*F. \tag{11}$$

Using the equation of motion for F, $F^* = m\phi^*$ and $F = -m\phi$ to eliminate the auxiliary field F, one obtains the scalar mass term

$$L_F = -m^2\phi^*\phi. \tag{12}$$

A non-zero anomalous MDM interaction would require the introduction of a new superfield term, $\Phi^* W^\alpha \Delta_\alpha \Phi$, to the Lagrangian.

$$L_m = ig\int d^2\theta \Phi^* W^\alpha \Delta_\alpha \Phi = -\frac{1}{2}g\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu} - ig\epsilon^{\mu\nu\lambda}F_{\mu\nu}\phi^* D_\lambda\phi$$
$$+ 2eg\bar{\lambda}\lambda\phi^*\phi - g\bar{\lambda}\gamma^\mu\psi(D_\mu\phi)^* - g\bar{\psi}\gamma^\mu\lambda(D_\mu\phi) - ig\bar{\psi}\lambda F + ig\bar{\lambda}\psi F^*. \tag{13}$$

Eliminating the auxiliary field we obtain the full Lagrangian

$$\begin{aligned} L = &-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}i\bar{\lambda}\gamma^\mu\partial_\mu\lambda \\ &+ \bar{\psi}i\gamma^\mu D_\mu\psi - m\bar{\psi}\psi + (D^\mu\phi)^*(D_\mu\phi) - m^2\phi^*\phi + ie\bar{\psi}\lambda\phi - ie\bar{\lambda}\psi\phi^* \\ &- \frac{1}{2}g\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu} - ig\epsilon^{\mu\nu\lambda}F_{\mu\nu}\phi^* D_\lambda\phi \\ &+ 2eg\bar{\lambda}\lambda\phi^*\phi - g\bar{\lambda}\gamma^\mu\psi(D_\mu\phi)^* - g\bar{\psi}\gamma^\mu\lambda(D_\mu\phi) \\ &- ig^2\bar{\psi}\lambda\bar{\lambda}\psi - igm\bar{\psi}\lambda\phi + igm\bar{\lambda}\psi\phi^*. \end{aligned} \tag{14}$$

The first term proportional to g in the above equation contains the usual anomalous MDM term $\bar{\psi}\sigma^{\mu\nu}\psi F_{\mu\nu}$ for a spinor with $g = \mu$ compared with Eq.(1). This term is responsible for the AC effect of a spin-1/2 particle. We identify the second term proportional to g in the above equation to be the corresponding AC effect term for a scalar particle.

To see the topological nature of these terms we note that they can be written as

$$L_{AC} = \frac{g}{2}F^{\mu\nu}\epsilon_{\mu\nu\lambda}j_F^\lambda - \frac{g}{2}F^{\mu\nu}\epsilon_{\mu\nu\lambda}j_S^\lambda, \tag{15}$$

where $j_F^\lambda = \bar{\psi}\gamma^\lambda\psi$, and $j_S^\lambda = i(\phi^* D^\lambda\phi - (D^\lambda\phi^*)\phi)$ are the current of the spinor and the scalar, respectively.

In general the interaction $F^{\mu\nu}\epsilon_{\mu\nu\lambda}j^\lambda$ generates a topological phase regardless of the specific value of the spin of the particle if the AC conditions required for the electric field is satisfied. This can be seen by studying the change of the action ΔS of the system due to

L_{AC}, for a closed trajectory from time 0 to time T for a point particle with velocity $\vec{v} \propto \vec{j}$. The topological phase generated for the spinor is given by [10]

$$\theta^F_{AC} = -\frac{1}{2} g \int_0^T F^{\mu\nu} \epsilon_{\mu\nu\lambda} j^\lambda_F = -g \int_0^T (\vec{S} \cdot \vec{v}) dt = -g \oint \vec{S} \cdot d\vec{r} = g\Lambda. \tag{16}$$

In the above $S_\mu = (1/2)\epsilon_{\mu\alpha\beta} F^{\alpha\beta}$. In the AC electric field configuration, $S_\mu = (0, E_2, E_1)$.

Similarly one obtains a topological phase θ^S_{AC} for the scalar when the AC conditions are satisfied with $\theta^S_{AC} = -g\Lambda$. We note that the topological phases developed for the spinor and scalar are the same in size and opposite in sign.

In the above discussions we have arbitrarily added the term of eq. (13), which contains a supersymmetric anomalous MDM interaction, to the Lagrangian of eq. (9), or equivalently eq. (10). It would be interesting to see if such an interaction can be naturally generated from some well known interactions, such as terms in eq. (9) itself.

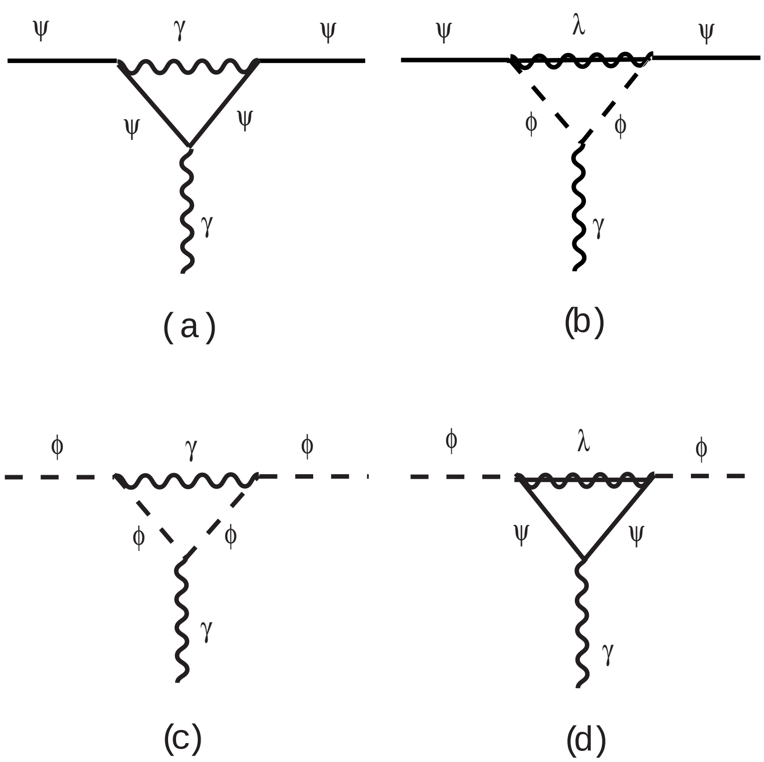

FIG. 1. Feynman diagrams for the anomalous MDM.

To this end we now study the radiative correction of spinor-photon, and scalar-photon interactions at quantum loop level starting with the Lagrangian in eq. (10). This theory does not have an anomalous MDM at tree level, but it can be generated at one loop level. The relevant diagrams are shown in Figure 1. Figure 1.a is the usual QED diagram generating an anomalous MDM for a spinor. Due to supersymmetric interactions, there is an additional diagram, Figure 1.b, contributing to the anomalous MDM for a spinor. Evaluating these two diagrams, we obtain an effective g in eq. (14) with

$$g = \frac{e^3}{16\pi m^2} \int_0^1 dx \int_y^1 \frac{y}{(y^2 - x(y-x)q^2/m^2)^{3/2}}, \tag{17}$$

where q is the photon momentum. Ferrara and Remiddi [11] showed that in 3+1 dimensions these two contributions cancelled, but we find that in 2+1 dimensions this cancellation does not happen.

The above result is logarithmically infrared divergent when q^2 approaches zero. This problem is similar to other infrared divergent problems in QED and can be regulated by introducing a small effective mass for the photon.

There are similar radiative corrections to scalar-photon couplings. These are shown in Figures 1.c and 1.d. Evaluating these diagrams, we indeed find the second term proportional to g in eq. (14) with the same g as in eq. (17) generated, as expected. The source for a non-zero contribution to g in this case is purely from Figure 1.d. This is quite different from the spinor case where Figures 1.a and 1.b both contribute to the anomalous MDM.

It is clear that the scalar AC effect in the above example is generated by the Yukawa coupling of a scalar interacts with two spinors, Figure 1.d. In 3+1 dimensions, such an interaction can not generate an AC effect for a scalar. This shows that the AC effect is intrinsically a 2+1 dimensional effect. Supersymmetry then provides the link between the spinor and scalar AC effects. With this understanding one can easily generate a topological AC effect for a scalar in a non-supersymmetric theory by introducing Yukawa couplings with at least one of the spinor having non-zero electric charge. This mechanism may be a realistic one to obtain scalar AC effect in some 2+1 dimensional systems.

To summarise we have studied the supersymmetric AC effect. In 3+1 dimensions, if supersymmetry is exact, no anomalous MDM interaction can exist. But in 2+1 dimensions, we find that a non-zero anomalous MDM interaction is possible. The related topological effect for a scalar is identified which is due to scalar current interaction with the dual of electric field. Since the Aharonov-Casher effect is essentially a phenomenon of two spatial dimensions, we conclude that there is an Aharonov-Casher effect for a spin-0 particle.

ACKNOWLEDGMENTS

This work was supported in part by the ROC NSC under grant number NSC 90-2112-M-002-014, by the ROC Ministry of Education Academic Excellence Project 89-N-FA01-1-4-3, and by the Australian Research Council. BHJMcK thanks the Department of Physics at the National Taiwan University for their hospitality where part of this work was done. XGH thanks the School of Physics at the University of Melbourne for their hospitality while this work was finalised.

REFERENCES

[1] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 485(1959).

[2] For a review see, M. Peshkin and A. Tonomura, The Aharonov-Bohm Effect (Springer-Verlag, Berlin, 1989).

[3] Y. Aharonov and A. Casher, Phys. Rev. Lett. **53**, 319(1984); See also J. Anandan, Phys. Rev. Lett. **48**, 1660(1982); Phys. Lett. **B138**, 347(1989).

[4] A. Cimmino et al., Phys. Rev. Lett. **63**, 380(1989); K. Sangster et al., Phys. Rev. Lett. **71**, 3641(1993).

[5] T. Boyer, Phys. Rev. **A36**, 5083(1987); Y. Aharonov, P. Pearle and L. Vaidman, Phys. Rev. **A37**, 4052(1988); A. Goldhaber, Phys. Rev. Lett. **62**, 482(1989); C. R. Hagen, Phys. Rev. Lett. **64**, 2347(1990).

[6] R. Mignani, J. Phys. **A24**, L421(1991) X.-G. He and B. H. J. McKellar, Phys. Lett. **B256**, 250(1991); ibid **B264**, 129(1991); X.-G. He, in Proceedings of " Test of Fundamental Laws in Physics", Les Arcs, Saviors, France, Jan 26 - Feb. 2, 1991, p352, Ed. O. Fackler et al., Frontières; C. Fuchs, Phys. Rev. **D42**, 2940(1990); B. Reznik, Phys. Rev. **D51**, 3108(1995); J. Maeda and K. Shizuya, Z. Phys. **C60**, 265(1993).

[7] Y. Azimov and R. Ryndin, JETP Lett. **61**, 345(1995); B. H. J. McKellar and J. A. Swansson, J. Phys. A **34** 1051 (2001), quant-ph/0007118.

[8] G. Spavieri, Phys. Rev. Lett. **81**, 1533 (1998); M. Wilkens, Phys. Rev. Lett. **81**, 1534 (1998); H. Liu, X. Y. Huang, and S. W. Qian, Chinese Phys. Lett. **12**, 327 (1995); J. Yi, G. S. Jeon, and M. Y. Choi, Phys. Rev. **B 52**, 7838 (1995); H. Q. Wei, R. S. Han, and X. Q. Wei, Phys. Rev. Lett. **75**, 2071 (1995); C. C. Chen, Phys. Rev. **A 51**, 2611 (1995); U. Leonhart and M. Wilkens, Europhys. Lett. **42**, 365 (1998); C. R. Hagen, Phys. Rev. Lett. **77**, 1656 (1996); H. Q. Wei, X. Q. Wei, R. S. Han, Phys. Rev. Lett. **77**, 1657 (1996).

[9] X.-G. He and B. H. J. McKellar, Phys. Rev. **A47**, 3424(1993); M. Wilkens, Phys. Rev. Lett. **72**, 5(1994); J. Dowling, C. Williams and J. Franson, Phys. Rev. Lett. **83**, 2486(1999); K. Kim and Y.-S. Tsai, Phys. Rev. **D7**, 3710(1978).

[10] X.-G. He and B. H.J. McKellar, quant-ph/0009065, Phys. Rev. **A64**, 022102(2001).

[11] S. Ferrara and E. Remiddi, Phys. Lett. **B53**, 347(1974).

[12] A. Salam and J. Stathdee, Phys. Rev. **D11**, 1521(1975); J. Wess and J. Bagger, supersymmetry and Supergravity (Princeton University Press, 1983); S.J. Gates et al., Superspace, or one thousand and one lessons in supersymmetry (Benjamin/Cummings, Menlo Park, 1983).